Letters

Comments on "Small-Signal Second-Harmonic Generation by a Nonlinear Transmission Line"

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In the above paper,[1] the important problem of second-harmonic frequency generation along nonlinear transmission lines is studied. However, some remarks concerning the fundamental role of dissipation and dispersion should be given which are already known from the literature.

First, in order to get large values of the efficiency, the attenuation has to be sufficiently small. Second, the dispersion has to be zero in the frequency band of consideration, whereas a cutoff frequency has to be established to prevent the flow of undesired harmonics ($> 2\omega$). This is a well-known and basic problem in, for example, nonlinear optics [1]. If, on the other hand, the dispersion is small and the higher harmonics can still be neglected, the amplitude of the second harmonic varies periodically with distance, the period being given by the coherence length [1], [2]. If the dispersion can be neglected in a larger frequency band, shock-wave phenomena will occur; this has been studied in numerous papers [2]–[5].

Practical nonlinear transmission lines on the basis of distributed Schottky-barrier diodes have already been proposed and studied [6], [7]. In contrast to Fig. 1 of the above paper,[1] the shunt admittance in that case is a series connection of the voltage-dependent capacitance and the bulk resistance. This circuit is not only a more physical description; it additionally introduces the desired dispersion and losses which increase with frequency so that the formation of shock waves is prevented (see also [8]). The relative importance and order of nonlinearity, dispersion, and dissipation have been discussed recently [7].

Equation (8) of the above paper[1] should read [7]

$$Q_1 = q'(V_0)V_1 + q''(V_0)V_2V_1^*$$

where the second term describes back mixing between the fundamental and the second harmonic to generate the fundamental. It is this term which is responsible for the periodicity of $V_2(z)$ leading to the coherence length. This contribution has been omitted in (8) and (9) of the above paper,[1] which is only valid if the influence of dissipation exceeds that of dispersion.

On the other hand, the results of the paper[1] are drawn from (18) by calculating (19) and considering (16). In that case, γ_n can be derived from (11) to yield

$$\alpha_n = \frac{1}{2}\sqrt{Lq'}\left(\frac{R}{L} + \frac{G}{q'}\right)$$

$$\beta_n = n\omega\sqrt{Lq'}\left[1 + \frac{1}{8n^2\omega^2}\left(\frac{R}{L} - \frac{G}{q'}\right)^2\right].$$

These expressions are valid up to second order in $R/\omega L$ and $G/\omega q'$.

Finally, instead of (20), we get

$$\phi \approx -j\alpha_1 z$$

which is purely imaginary. Physically, the equivalent circuit of Fig. 1 of the above paper[1] yields zero dispersion and finite losses, so a coherence length cannot be introduced. As a result, wave propagation is determined by nonlinearity and dissipation in a first order and shock-wave phenomena can occur which, however, depend on the size of the losses [2].

Reply[2] *by K. S. Champlin and D. R. Singh*[3]

The authors appreciate Prof. Jäger's comments concerning the roles of dissipation and dispersion in determining the efficiency of second harmonic generation. We agree that optimum efficiency will occur with dispersion zero, attenuation small, and undesired harmonics suppressed. We would like to point out, however, that the model of Fig. 1 was not intended to represent any particular implementation or to necessarily contain frequency-independent elements. It was used merely to describe a generic transmission line in a convenient fashion. If the transmission line elements are assumed independent of frequency, then with low loss the dispersion is zero and Prof. Jäger's comments are indeed correct.

Our paper was not presented as a rigorous treatment of any particular line but as a simple derivation of a useful "figure of merit" for comparing the SHG potential of different transmission-line implementations. The authors still believe the (Q_2K_N) product to be such a figure of merit and stand by the original conclusions of the paper.

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Manuscript received June 10, 1986.
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IEEE Log Number 8611016.

[1]K. S. Champlin and D. R. Singh, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 351–353, Mar. 1986.

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IEEE Log Number 8611017.

0018-9480/87/0100-0076$01.00 ©1987 IEEE